Exercise 28

Evaluate the integral.

$$\int_0^4 (4-t)\sqrt{t}\,dt$$

Solution

According to part 2 of the fundamental theorem of calculus,

$$\int_a^b f(x)\,dx = F(b) - F(a),$$

where F is an antiderivative of f. Use the power rule in reverse here: Bump up the exponent by 1 and divide by that exponent.

$$\begin{aligned}
\int_0^4 (4-t)\sqrt{t}\,dt &= \int_0^4 (4-t)t^{1/2}\,dt \\
&= \int_0^4 (4t^{1/2} - t^{3/2})\,dt \\
&= \int_0^4 4t^{1/2}\,dt - \int_0^4 t^{3/2}\,dt \\
&= \left(\frac{4t^{3/2}}{\frac{3}{2}}\right)\Bigg|_0^4 - \left(\frac{t^{5/2}}{\frac{5}{2}}\right)\Bigg|_0^4 \\
&= \frac{8}{3}(t^{3/2})\Big|_0^4 - \frac{2}{5}(t^{5/2})\Big|_0^4 \\
&= \frac{8}{3}(4^{3/2} - 0^{3/2}) - \frac{2}{5}(4^{5/2} - 0^{5/2}) \\
&= \frac{8}{3}(8) - \frac{2}{5}(32) \\
&= \frac{128}{15}
\end{aligned}$$